Secret price cuts, or:

Price coordination when supervising the partners is difficult

Own demand observable
Market demand *not* observable
Other firms' prices *not* observable

When own demand is low, is it because market demand is low, or because partners default?

Punishment ($p = c$) is necessary.
But punishment forever?

Can firms coordinate prices without being able to observe each other's prices?

Punishment starts when one observes low demand.
Punishment phase lasts for a finite number of periods.
Even colluding firms have periods of ''price wars''.

Model: Two firms; homogeneous products; $MC = c$.

In each period: firms set prices; consumers choose the firm with the lowest price.

Market demand is either:
$D = 0$, with probability α;
$D = D(p)$, with probability $(1 - \alpha)$.

Both firms know it if at least one firm has zero profit in a period. Either:

- market demand is zero and both firms have zero profit, or
- one firm has cut its price and knows that the other firm has zero profit

Strategy:

- Start with $p = p^m$.
- Set $p = p^m$ until (at least) one firm has zero profit.
- If this happens, then set $p = c$ for T periods.
- After T periods, return to $p = p^m$ until (at least) one firm has zero profit.

Is there an equilibrium in which each firm plays this strategy?

T must be determined.

Two phases:

- Colluding phase
- Punishment phase

V^+ = net present value of a firm in the colluding phase

V^- = net present value of a firm *at the start of* the punishment phase

$$V^+ = (1-\alpha)\left(\frac{\pi^m}{2} + \delta V^+\right) + \alpha\delta V^-$$

$$V^- = \delta^T V^+$$

Equilibrium condition:

$$V^+ \geq (1-\alpha)(\pi^m + \delta V^-) + \alpha\delta V^- = (1-\alpha)\pi^m + \delta V^-$$

$$\Leftrightarrow (1-\alpha)\left(\frac{\pi^m}{2} + \delta V^+\right) + \alpha\delta V^- \geq (1-\alpha)\pi^m + \delta V^-$$

$$\Leftrightarrow \delta\left(V^+ - V^-\right) \geq \frac{\pi^m}{2}$$

$$\Leftrightarrow V^+\delta\left(1-\delta^T\right) \geq \frac{\pi^m}{2}$$

$$V^+ = (1-\alpha)\left(\frac{\pi^m}{2} + \delta V^+\right) + \alpha\delta^{T+1}V^+$$

$$V^+ = \frac{(1-\alpha)\frac{\pi^m}{2}}{1-(1-\alpha)\delta - \alpha\delta^{T+1}}$$

$$\frac{(1-\alpha)\frac{\pi^m}{2}}{1-(1-\alpha)\delta - \alpha\delta^{T+1}}\delta(1-\delta^T) \geq \frac{\pi^m}{2}$$

$$2\delta(1-\alpha) + (2\alpha - 1)\delta^{T+1} \geq 1$$

The best equilibrium has the highest possible V^+.

The firms' problem:

$\max_T\ V^+$, such that: $2\delta(1-\alpha) + (2\alpha - 1)\delta^{T+1} \geq 1$

But: $\mathrm{d}V^+/\mathrm{d}T < 0$. So we restate the problem.

$\min T$, such that: $2\delta(1-\alpha) + (2\alpha - 1)\delta^{T+1} \geq 1$

$T = 0$ is too low – there has to be some punishment, even under collusion:

$$2\delta(1 - \alpha) + (2\alpha - 1)\delta = \delta < 1$$

And the lefthand side must be increasing in T:

$$\frac{d}{dT}\left[2\delta(1-\alpha)+(2\alpha-1)\delta^{T+1}\right]$$

$$=(2\alpha-1)\delta^{T+1}\underbrace{\ln\delta}_{<0} > 0 \Leftrightarrow \alpha < \frac{1}{2}$$

If $\alpha \geq ½$, then collusion is impossible: The probability of zero market demand is too large.

If $\alpha < ½$, then $2\alpha - 1 < 0$. But $(2\alpha - 1)\delta^{T+1} \rightarrow 0$ as $T \rightarrow \infty$.

Equilibrium condition satisfied for some T if also

$$2\delta(1 - \alpha) \geq 1$$

All in all: Collusion can occur in equilibrium if:

- $\alpha < ½$
- $\delta \geq \frac{1}{2}\frac{1}{1-\alpha}$

T is chosen as the lowest integer that satisfies:

$$2\delta(1 - \alpha) + (2\alpha - 1)\delta^{T+1} \geq 1$$

Example: $\delta = ¾$, $\alpha = ¼$. Condition: $(¾)^{T+1} \leq ¼ \Rightarrow T^* = 4$.
But often T^* is smaller: $\delta = 0.9$, $\alpha = 0.2 \Rightarrow T^* = 2$.

Price rigidities

- Menu costs
- Price reactions not punishments, but attempts to regain market share

Suppose
- a price is fixed for two periods
- firms alternate at setting price

Duopoly with alternating price setting

- A discrete price grid

- *Markov strategies*: strategies based only on directly payoff-relevant information

Example: A trigger strategy is *not* Markov; no price from the past has a *direct* effect on a firm's profit today, only an *indirect* effect, because other firms use trigger strategies.

A restriction to Markov strategies would be too strong when moves are simultaneous. Here, moves are alternating.

Model: duopoly; each firm's price fixed for two periods; firm 1 sets price in odd-numbered periods (1 – 3 – 5 – …), firm 2 in even-numbered periods (2 – 4 – 6 – …).

Markov reaction functions:

Let p_{it} be the price set by firm i in period t.

Firm 1’s reaction function:

$$p_{1,\,2k+1} = R_1(p_{2,\,2k}), \quad k = 0, 1, 2, \ldots$$

Firm 2’s reaction function:

$$p_{2,\,2k+2} = R_2(p_{1,\,2k+1}), \quad k = 0, 1, 2, \ldots$$

Markov perfect equilibrium: An equilibrium in Markov reaction functions. At the start of each subgame, the firm that makes the move chooses an optimum strategy, given the restriction only to pay attention to payoff-relevant information, and given the other firm’s equilibrium strategy.

The two firms at any point in time:
‘‘the active” and ‘‘the other”

Consider the active firm’s decision today.

Suppose the other firm set the price p_h last period; this is also its price today. – We are in state h.

V_h – the active firm’s net present value in state h.
W_h – the other firm’s net present value in state h.

Tomorrow, the roles are changed.

Profit per period: π(own price, the other's price)

$$\Rightarrow \; V_h = \max_k \left[\pi\left(p_k, p_h\right) + \delta W_k \right]$$

<u>A symmetric equilibrium:</u> $R_1(\cdot) = R_2(\cdot) = R(\cdot)$

<u>Mixed strategy:</u> A firm may be indifferent between one or more prices, and in equilibrium, the other firm has beliefs about which of these prices will be chosen. These beliefs will then constitute the firm's mixed strategy.

α_{hk} – the probability (according to the other firm's beliefs) that a firm in state h chooses price p_k.

Note: $\sum_k \alpha_{hk} = 1$

A symmetric equilibrium can be described by a *transition matrix*: Suppose there are H possible prices.

$$\text{from state } h \left\{ \begin{bmatrix} \alpha_{11} & \cdots & \cdots & \alpha_{1H} \\ . & & & . \\ . & & & . \\ . & & & . \\ \alpha_{H1} & \cdots & \cdots & \alpha_{HH} \end{bmatrix} \right. = A$$

$$\underbrace{\hphantom{\alpha_{11} \quad \cdots \quad \cdots \quad \alpha_{1H}}}_{\text{to state } k}$$

Equilibrium conditions

$$V_h = \sum_k \alpha_{hk} \left[\pi(p_k, p_h) + \delta W_k \right]$$
$$W_k = \sum_l \alpha_{kl} \left[\pi(p_k, p_l) + \delta V_l \right]$$

These are the values of V_h and W_k that follow from the transition matrix A.

$[V_h - \pi(p_k, p_l) - \delta W_k]\alpha_{hk} = 0, \ \forall\ h, k.$

$V_h \geq \pi(p_k, p_l) + \delta W_k, \ \forall\ h, k.$

Complementary slackness: If $\alpha_{hk} > 0$, it must be because $V_h = \pi(p_k, p_l) + \delta W_k$, that is, because p_k maximizes the firm's net present value in state h.

$$\sum_k \alpha_{hk} = 1, \ \forall\ h$$

$\alpha_{hk} \geq 0, \ \forall\ h, k.$

Example:

$D(p) = 1 - p$; $c = 0$

The price grid: $p_h = \frac{h}{6}$, $h = 0, \ldots, 6$.
Competitive price: $p_0 = 0$. Monopoly price: $p_m = p_3 = ½$.

Two (symmetric Markov perfect) equilibria (at least):

1. ‘‘Kinked demand curve”: The other firm does *not* follow you if you increase the price but undercuts you if you decrease the price.

$R(1) = R(\frac{5}{6}) = R(\frac{2}{3}) = R(\frac{1}{2}) = R(0) = \frac{1}{2}$;
$R(\frac{1}{3}) = \frac{1}{6}$; $R(\frac{1}{6}) \in \{\frac{1}{6}, \frac{1}{2}\}$.

- Either the game starts in state 3 and stays there, or it ends there sooner or later (absorbing state).

- A mixed strategy in state 1 – a waiting game (‘‘war of attrition”): Each firm is indifferent between meeting p_1 with p_1, and making a short-term sacrifice in order to get the monopoly price from next period on.

- The equilibrium is sustainable only if each firm is able to supply the whole market demand at $p_1 = \frac{1}{6}$: $D(\frac{1}{6}) = \frac{5}{6}$. In the absorbing state 3, each firm sells $\frac{1}{2}D(p_3) = \frac{1}{4}$ but needs to keep an excess capacity of $\frac{5}{6} - \frac{1}{4} = \frac{7}{12}$.

<u>2. Price war:</u> The firms undercut each other.

$R(1) = R(\frac{5}{6}) = \frac{2}{3}$; $R(\frac{2}{3}) = \frac{1}{2}$; $R(\frac{1}{2}) = \frac{1}{3}$;
$R(\frac{1}{3}) = \frac{1}{6}$; $R(\frac{1}{6}) = 0$; $R(0) \in \{0, \frac{5}{6}\}$.

- Unstable prices: no absorbing state.
- Edgeworth cycle.
- Again a waiting game. But now the price jumps beyond the monopoly price.

*

- Multiple equilibria, even when we restrict attention to Markov strategies.
- Fewer equilibria than in an ordinary repeated game.
- $p = c$ is no longer an equilibrium; there is always *some* price collusion in equilibrium.